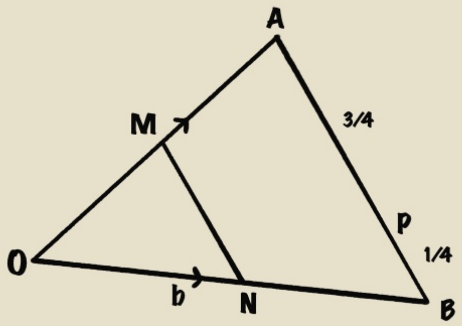
The Maths Society



In the diagram

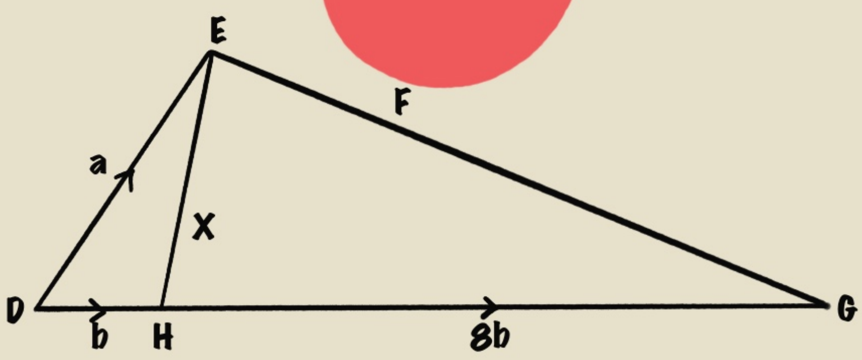
$\overrightarrow{DE} = a$

$\overrightarrow{DH} = b$

$\overrightarrow{HG} = 8b$

$EX : XH = 3 : 1$

$EF : FG = 1 : 3$



17 **(a)** **(i)** $\mathbf{m} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$

Find 3**m**.

$\begin{pmatrix} 15 \\ 21 \end{pmatrix}$ [1]

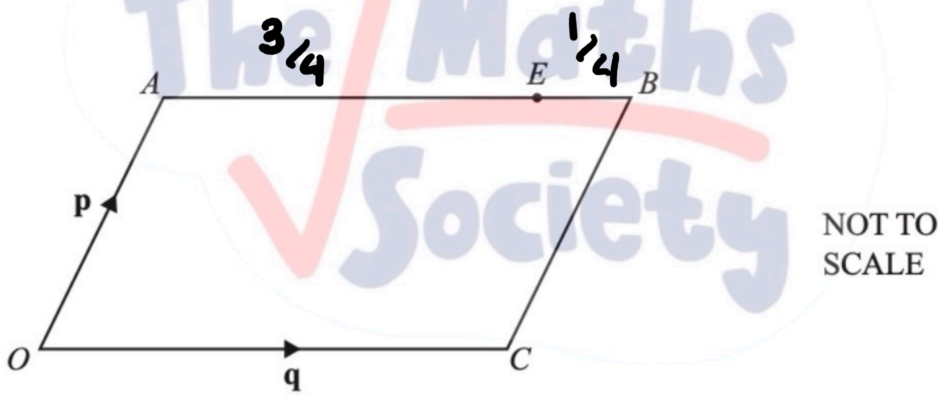
(ii) $\overrightarrow{VW} = \begin{pmatrix} 10 \\ -24 \end{pmatrix}$

Find $|\overrightarrow{VW}|$.

$\sqrt{(10)^2 + (-24)^2} = 2\,40$

.... 26 .... [2]

(b)



OABC is a parallelogram.
$\overrightarrow{OA} = \mathbf{p}$ and $\overrightarrow{OC} = \mathbf{q}$.
E is the point on *AB* such that *AE* : *EB* = 3 : 1.

Find $\overrightarrow{OE}$, in terms of **p** and **q**, in its simplest form.

$\overrightarrow{OE} = \overrightarrow{OA} + \overrightarrow{AE}$

$= \underline{p} + \frac{3}{4} q$

$\overrightarrow{OE} =$ ...... $\underline{p} + \frac{3}{4}\underline{q}$ ...... [2]

2 **(a)** $\mathbf{p} = \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ $\mathbf{q} = \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

(i) Find $2\mathbf{p} + \mathbf{q}$.

$2\begin{pmatrix} 4 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix} = \begin{pmatrix} 8 \\ 10 \end{pmatrix} + \begin{pmatrix} -2 \\ 7 \end{pmatrix}$

$= \begin{pmatrix} 6 \\ 17 \end{pmatrix}$

$\begin{pmatrix} 6 \\ 17 \end{pmatrix}$ [2]

(ii) Find $|\mathbf{p}|$.

$\sqrt{4^2 + 5^2} = \sqrt{41}$

$= 6.4$

6.4 [2]

(b) A is the point (4, 1) and $\overrightarrow{AB} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}$.

Find the coordinates of B.

(1 , 2) [1]

(c) The line $y = 3x - 2$ crosses the y-axis at G.

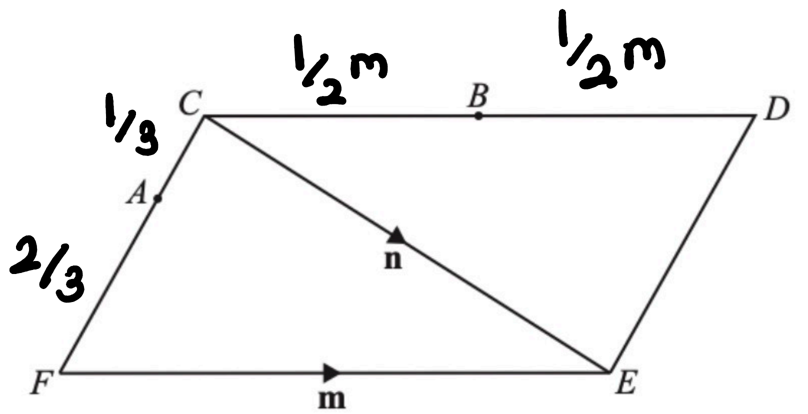
Write down the coordinates of G.

$y = 3(0) - 2$

$= -2$

(0 , -2) [1]

23 (a)



NOT TO SCALE

The diagram shows a parallelogram $CDEF$.
$\overrightarrow{FE} = \mathbf{m}$ and $\overrightarrow{CE} = \mathbf{n}$.
B is the midpoint of CD.
$FA = 2AC$

Find an expression, in terms of **m** and **n**, for $\overrightarrow{AB}$.
Give your answer in its simplest form.

$\overrightarrow{CF} = \underline{n} - \underline{m}$

$\overrightarrow{AB} = \frac{1}{3}\underline{n} - \frac{1}{3}\underline{m} + \frac{1}{2}\underline{m}$

$= \frac{1}{3}\underline{n} + \frac{1}{6}\underline{m}$

$\overrightarrow{AB} =$ $\frac{1}{3}\underline{n} + \frac{1}{6}\underline{m}$ [3]

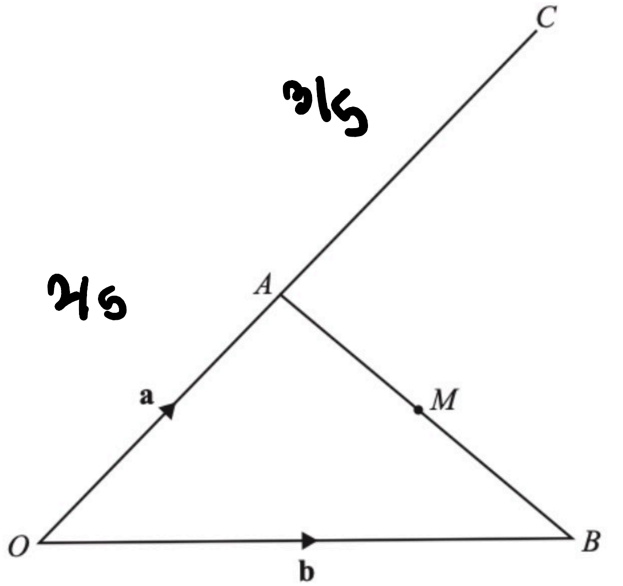
(b) $\overrightarrow{GH} = \frac{5}{6}(2\mathbf{p} + \mathbf{q})$ $\qquad \overrightarrow{JK} = \frac{5}{18}(2\mathbf{p} + \mathbf{q})$

Write down **two** facts about vectors $\overrightarrow{GH}$ and $\overrightarrow{JK}$.

They are parallel to each other

$\overrightarrow{GH}$ has a larger magnitude [2]

22



NOT TO SCALE

The diagram shows a triangle OAB and a straight line OAC.
$OA : OC = 2 : 5$ and M is the midpoint of AB.
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$.

Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, in its simplest form

(a) $\overrightarrow{AB}$,

$\overrightarrow{AB} =$ $\underline{b} - \underline{a}$ .... [1]

(b) $\overrightarrow{MC}$.

$\overrightarrow{AB} = \underline{b} - \underline{a}$

$\overrightarrow{MA} = \frac{1}{2}\underline{b} - \frac{1}{2}\underline{a}$

$\frac{2}{5}x = a$

$x = \frac{5}{2}a$

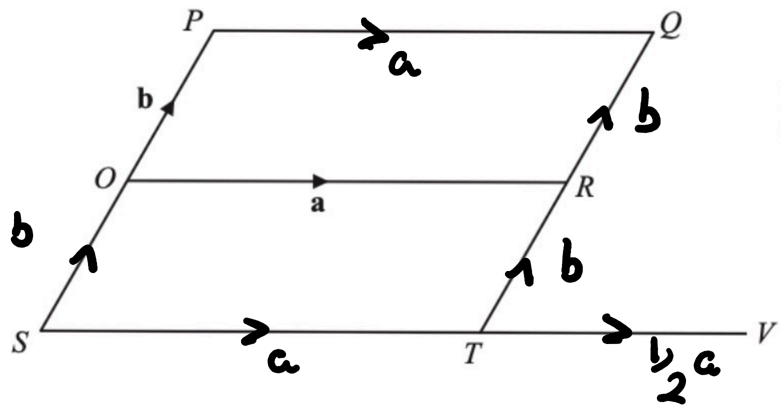
$\frac{3}{5}x = \frac{3}{2}\underline{a}$

$\overrightarrow{MC} = \frac{1}{2}\underline{b} - \frac{1}{2}\underline{a} + \frac{3}{2}\underline{a}$

$= \frac{1}{2}\underline{b} + \underline{a}$

$\overrightarrow{MC} =$ $\frac{1}{2}\underline{b} + \underline{a}$ .... [3]

21



O is the origin and $OPQR$ is a parallelogram.
SOP is a straight line with $SO = OP$.
TRQ is a straight line with $TR = RQ$.
STV is a straight line and $ST : TV = 2 : 1$.
$\overrightarrow{OR} = \mathbf{a}$ and $\overrightarrow{OP} = \mathbf{b}$.

(a) Find, in terms of **a** and **b**, in its simplest form,

(i) the position vector of T,

$\overrightarrow{OT} = \underline{a} - \underline{b}$

$\underline{a} - \underline{b}$ .... [2]

(ii) $\overrightarrow{RV}$.

$\overrightarrow{RV} = \frac{1}{2}\underline{a} - \underline{b}$ .... [1]

(b) Show that PT is parallel to RV.

$\overrightarrow{PT} = -2\underline{b} + \underline{a}$

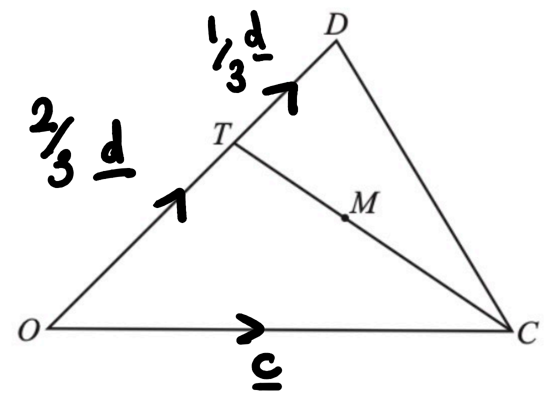
$= \underline{a} - 2\underline{b}$

$= 2\left(\frac{1}{2}\underline{a} - \underline{b}\right)$

$\therefore \overrightarrow{PT} = 2\overrightarrow{RV}$

[2]

(d)



NOT TO SCALE

In the diagram, O is the origin, $OT = 2TD$ and M is the midpoint of TC.
$\overrightarrow{OC} = \mathbf{c}$ and $\overrightarrow{OD} = \mathbf{d}$.

Find the position vector of M.
Give your answer in terms of $\mathbf{c}$ and $\mathbf{d}$ in its simplest form.

$$\overrightarrow{TC} = \overrightarrow{OC} - \overrightarrow{OT}$$

$$= \underline{c} - \tfrac{2}{3}\underline{d}$$

$$\overrightarrow{TM} = \tfrac{1}{2}\underline{c} - \tfrac{1}{3}\underline{d}$$

$$\overrightarrow{OM} = \overrightarrow{OT} + \overrightarrow{TM}$$

$$= \tfrac{2}{3}\underline{d} + \tfrac{1}{2}\underline{c} - \tfrac{1}{3}\underline{d}$$

$$= \tfrac{1}{3}\underline{d} + \tfrac{1}{2}\underline{c}$$

$\frac{1}{3}\underline{d} + \frac{1}{2}\underline{c}$ .... [3]

8 **(a)** $\overrightarrow{AB} = \begin{pmatrix} 6 \\ -1 \end{pmatrix}$ $\overrightarrow{BC} = \begin{pmatrix} -2 \\ 5 \end{pmatrix}$ $\overrightarrow{DC} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

Find

(i) $\overrightarrow{AC}$,

$\overrightarrow{AC} = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ [2]

(ii) $\overrightarrow{BD}$,

$\overrightarrow{DC} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

$\therefore \overrightarrow{CD} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$\overrightarrow{BD} = \begin{pmatrix} -4 \\ 8 \end{pmatrix}$ [2]

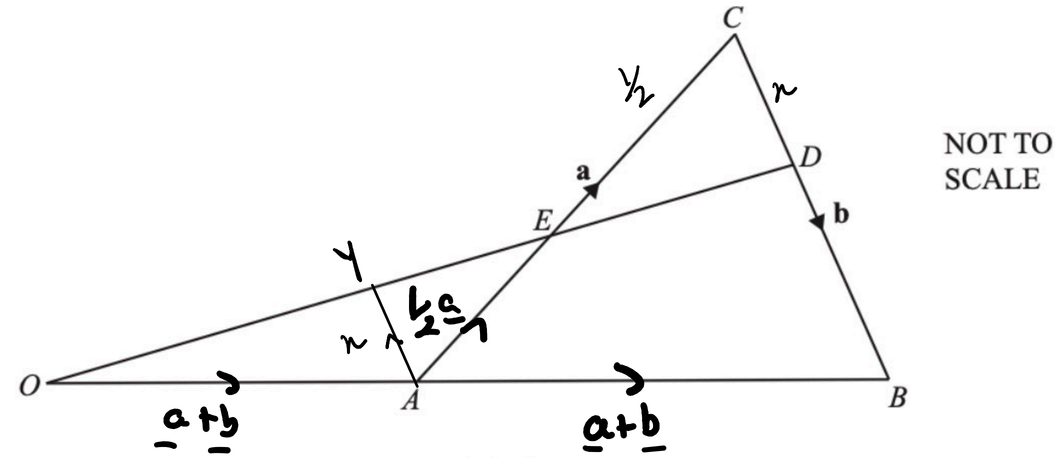
(iii) $|\overrightarrow{BC}|$.

$\sqrt{(-2)^2 + (5)^2}$

$= \sqrt{29} = 5.39$

5.39 [2]

(b)



In the diagram, OAB and OED are straight lines.
O is the origin, A is the midpoint of OB and E is the midpoint of AC.
$\overrightarrow{AC} = \mathbf{a}$ and $\overrightarrow{CB} = \mathbf{b}$.

Find, in terms of **a** and **b**, in its simplest form

(i) $\overrightarrow{AB}$,

$\overrightarrow{AB} =$ $\underline{a} + \underline{b}$ .... [1]

(ii) $\overrightarrow{OE}$,

$\overrightarrow{AE} = \frac{1}{2}\underline{a}$

$\overrightarrow{OE} = \underline{a} + \underline{b} + \frac{1}{2}\underline{a}$

$\overrightarrow{OE} =$ $\frac{3}{2}\underline{a} + \underline{b}$ .... [2]

(iii) the position vector of D.

$\overrightarrow{OD} = \overrightarrow{OE} + \overrightarrow{EC} + \overrightarrow{CD}$

$= \frac{3}{2}\underline{a} + \underline{b} + \frac{1}{2}\underline{a} + \overrightarrow{CD}$

$= 2\underline{a} + \underline{b} + \overrightarrow{CD}$

$= 2\underline{a} + \underline{b} + \frac{1}{3}\underline{b}$

$2a + \frac{4}{3}\underline{b}$ .... [3]

$x = \frac{1}{2}\overline{DB}$

$\overrightarrow{CD} = \frac{1}{2}\overrightarrow{DB}$

$\frac{1}{2}\overrightarrow{DB} + \overrightarrow{DB} = \underline{b}$

$\overrightarrow{DB} = \frac{2}{3}\underline{b}$ $\therefore$ $\overrightarrow{CD} = \frac{1}{3}\underline{b}$